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A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension Insert

Practice Paper – Set 3

Time allowed: 2 hours

INFORMATION FOR CANDIDATES

- This insert contains the article for Section B.
- This document consists of **4** pages. Any blank pages are indicated.

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The mathematics of price and demand

When businesses make decisions about the price to sell their products for, they need to take a number of factors into account. One important consideration is the cost of producing the product; the business needs to make a profit. Another consideration is how many they are likely to sell; the term used for this is the *quantity demanded* or, more briefly, *demand*. The demand for a product usually goes down as the price increases. It may be more cost-effective to sell at a lower price; this will allow production of more of the product and it may be cheaper to mass-produce.

Prices change over time, so it is of interest to businesses to know what effect changing a price is likely to have. This is also of interest to governments which tax some products; a change in tax will result in a change in price – if this reduces demand, the total income from tax could go down.

Demand curves

Demand curves are used to show how the demand for a product changes as the price changes; price is usually shown on the vertical axis with demand for the product on the horizontal axis. In simple models, the demand curve is often assumed to be a straight line, as in the example in Fig. C1 below.

Price elasticity of demand (PED)

The effect of an increase in price on the demand for a product is described by the *price elasticity of demand* (PED). For a straight line demand curve, price elasticity of demand is the percentage increase in demand for a 1% increase in price; it can be calculated using the following formula.

$$\text{Price elasticity of demand} = \frac{\%\text{ increase in quantity demanded}}{\%\text{ increase in price}}$$

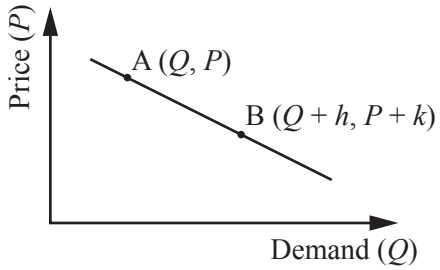

Fig. C1

For a straight line demand curve, as shown in Fig. C1, at the point A, with coordinates (Q, P), the percentage increase in demand when moving to point B, with coordinates $(Q+h, P+k)$ is $\frac{100h}{Q}$.

Using the corresponding expression for the percentage increase in price, the price elasticity of demand at A simplifies to $\frac{P}{mQ}$ where m is the gradient of the straight line. The gradient m of the line is constant, but the value of $\frac{P}{Q}$ is different at different points on the line, so the value of $\frac{P}{mQ}$ also varies i.e. when demand is modelled by a straight line, the PED varies.

PED was first proposed as a measure of how responsive demand is to changes in price by the economist Alfred Marshall in 1890. For most things that people buy, demand decreases as price increases, so the demand curve has a negative gradient and the PED is negative also. The negative sign of the PED is however often ignored, so that if demand changes a lot when there is a change in price, the PED is said to be 'high' while in situations where demand is not very sensitive to price changes the PED is said to be 'low'.

Calculating revenue

The total income (or *revenue*) from selling the product will vary as demand and price change.

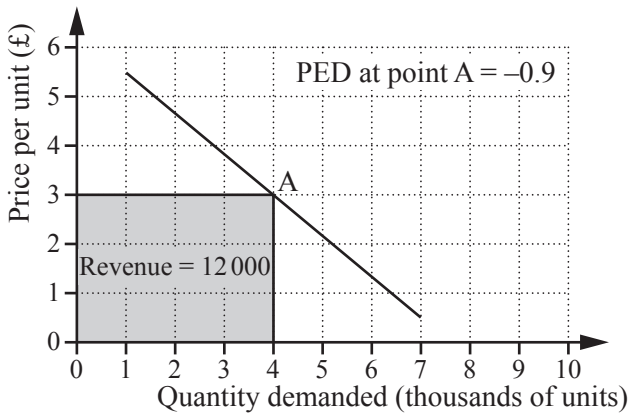

Fig. C2

As shown in Fig. C2, the revenue is calculated by multiplying the quantity demanded by the price. The revenue is represented by the area of a rectangle and it can be shown that the maximum revenue occurs when the point A on the line is the point with PED equal to −1.

Constant PED

Sometimes the way that demand changes with price is better modelled by a curve than by a straight line. Fig. C3 shows a typical demand curve, one in which higher demand is associated with lower price.

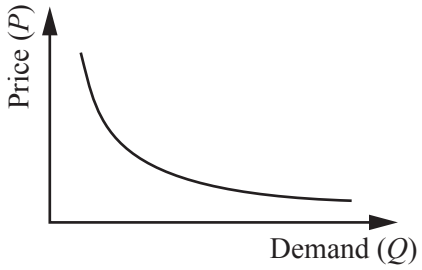

Fig. C3

The gradient of the curve is $\frac{dP}{dQ}$ and the PED at a point on the curve is defined to be $\frac{dQ}{dP}\frac{P}{Q}$. This corresponds to the formula $\frac{P}{mQ}$ for the straight line demand curve.

For certain curves, the PED has the same value at all points of the curve. If the PED has the constant value k, then $k = \frac{dQ}{dP}\frac{P}{Q}$, and this can be rearranged as $\frac{dP}{dQ} = \frac{1}{k}\frac{P}{Q}$.

When PED is constant, a given percentage change in price results in the demand changing by a percentage which is independent of the original price.